Heat transfer to a continuous moving flat surface with variable wall temperature

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Transient heat transfer from a continuous moving flat surface with varying wall temperature is studied. Numerical results are presented for the transient temperature profiles and heat transfer rates from the wall for Prandtl numbers varying from 0.01 to 1000. Asymptotic solutions for steady state heat transfer rates for large Prandtl number are also presented.

Keywords: continuous moving plate; transient and steady-state heat transfer; varying wall temperature

Introduction

The study of heat transfer or mass transfer to and from a continuous flat surface moving at high speed is of considerable practical interest. Such systems are used in the fabrication of sheet glass, steel plates, paper drying, electrotinning of steel sheets and copper wire, hot rolling, hot extrusion, cold extrusion and continuous casting. Sakiadis[1–3] was the first to study this class of boundary layer problem, where a numerical solution was obtained for two-dimensional flow induced by a long moving plate or cylinder using similarity transformation. Experimental investigations of the flow field were made by Tsou *et al.*[4] and Griffith.[5] Numerical solutions of the steady-state thermal boundary layers on the continuous flat surface have been obtained by Tsou *et al.*,[4] Rhodes and Kaminer,[6] Erickson *et al.*,[7] and by Rotte and Beek,[8] Bourne and Elliston,[9] and Karmis and Pechoc[10] in case of cylindrical surfaces. In these works the thickness of the plate or material was considered to be negligibly small as compared to the distance along the surface. Griffin and Throne[11] have reported an experimental study of heat transfer from a continuously moving belt in air. Their results were in agreement with the theoretical results of Erickson *et al.*[7]

However, in cases such as in continuous casting, the thickness of the emerging plate is finite; hence, one has to consider the conduction within the plate. Karwe and Jaluria[12,13] have included the conjugate transport resulting from conduction within finite size plate while analyzing the heat transport from a continuous moving plate. In case of continuous extrusion of the polymer from a die, the thin polymer sheet or filament constitute a continuous moving solid with a nonuniform surface velocity and temperature.[14] Soundalgekar and Murty[15] used power law surface temperature to investigate steady-state heat transfer from a continuous moving surface. Jeng *et al.*[16] further considered orbitrary surface velocity and nonuniform surface temperature for this problem.

As the analysis of the boundary layer near continuously moving surface is similar for the cases of heat transfer and mass transfer, the results obtained for heat transfer characteristics can be used in case of mass transfer by replacing the Prandtl and Nusselt numbers respectively by Schmidt and Sherwood numbers. Chin[17] presented an asymptotic solution valid for large Schmidt numbers for mass transfer to a continuously moving plate under laminar conditions. Gorla has studied the transient mass transfer to a continuous moving plate with step change in surface concentration[18] and with step change in surface mass flux[19] using similarity transformation. These results can be used for heat transfer case with uniform wall temperature and uniform heat flux condition.

As this problem is of interest to both heat transfer and mass transfer cases, the present note considers transient heat transfer from continuous moving plate with step change in variable wall temperature. The variation in wall temperature considered is $T_w - T_\infty = Ax^n$, where A is constant. For such a power law variation on wall temperature or mass concentration, the similarity formulation holds good.[19,20] The results are presented for a range of Prandtl number from 0.01 to 1000 and $n>0$. In the formulation it is assumed that the plate thickness is negligibly small compared with its length, hence the conduction within the body of the plate is neglected. In the case of the plate being heated from the ambient fluid, the surface temperature can be well represented by the power law variation from the leading edge, assuming constant surface heat transfer coefficient. The present results are also useful for the case of mass transfer to the plate such as in electroplating.[17] Flows with large Prandtl number may result in chemical processing of hydrocarbons and silicone polymers.[21] Also, large Schmidt number is encountered in mass transfer case. Hence, asymptotic steady state solutions at large Prandtl number are also presented.

Transient solution

The momentum and energy equations governing the heat transfer from a continuously moving plate whose variable surface temperature undergoes step change with time are similar to those equations given by Gorla[19] in case of transient mass transfer to a continuous moving sheet electrode. These equations in nondimensional form after similarity transformation are given as

$$f''' + \frac{ff''}{2} = 0 \tag{1}$$

$$Pr(1 - f'\tau)\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\eta^2} + \frac{Prf}{2}\frac{\partial\theta}{\partial\eta} - nPrf'\theta \tag{2}$$

The initial condition is $\theta(\eta, 0) = 0$ and the boundary conditions are $f(0)=0$, $f'(0)=1$, $f'(\infty)=0$, $\theta(0,\tau)=1(\tau)$ and $\theta(\infty,\tau)=0$. Here the prime denotes the differentiation with respect to η. The solutions for the velocity profiles are known.[4] The energy

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Notation

$f(\eta)$	Nondimensional stream function
h	Local heat transfer coefficient
k	Thermal conductivity
n	Surface temperature variation parameter
Nu	Nusselt number, hx/k
Pr	Prandtl number, ν/α
Re	Local Reynolds number, $u_w x/\nu$
t	Time
T	Temperature
u	Velocity component in x direction
v	Velocity component in y direction
x	Coordinate along the moving surface
y	Coordinate normal to the surface

Greek letters

α	Thermal diffusivity
β	Beta function
ζ	Nondimensional stretched coordinate, $Pr^{1/2}\eta$
η	Nondimensional coordinate, $yu_w/\nu x^{1/2}$
θ	Nondimensional temperature, $(T-T_\infty)/(T_w-T_\infty)$
θ_n	Functions of ζ ($n=0, 1, 2$)
ν	Kinematic viscosity
τ	Nondimensional time, $u_w t/x$

Subscripts

0, 1, 2	Zero, first, second order solutions defined in Equation 3
s	Steady-state
w	Condition at the wall
∞	Conditions at very large distances away from the wall

equation (Equation 2) was solved using Crank–Nicholson type implicit finite difference scheme.[22] The present solution at $n=0$, which corresponds to uniform wall temperature, agreed with those obtained by Gorla[18] in case of mass transfer. The growth of the thermal boundary layer with time are shown in Figures 1–4 for Prandtl numbers 0.01, 0.7, 7 and 1000 respectively. From these figures we observe that the temperature profiles monotonically increase to their steady-state shape. With increase in time, the thermal boundary layer thickness decreases with increase in n. At the start of the transient the effect of wall temperature variation on the thermal boundary is small, especially for small Prandtl number, and with increase in time the effect is more predominant. Of practical interest, the steady-state heat transfer rates are given in Table 1 in terms of $Nu/\mathrm{Re}^{1/2} = -\theta'(0, \infty)$. In Figure 5, the ratio of the instantaneous to steady-state values of heat transfer rates are shown for Pr 0.01, 0.7, 7 and 1000 for different n values. We observe that the heat transfer rate at the surface approaches the steady-state condition asymptotically. The steady-state is reached at early times with increase in the value of n. The time to reach steady-state increases with increase in Prandtl number.

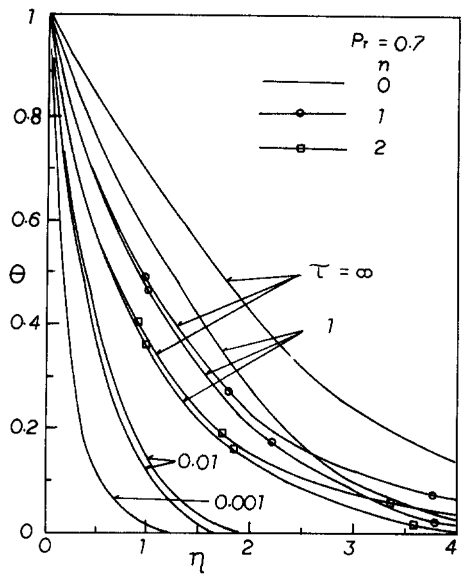

Figure 2 Transient temperature profiles for $Pr=0.7$

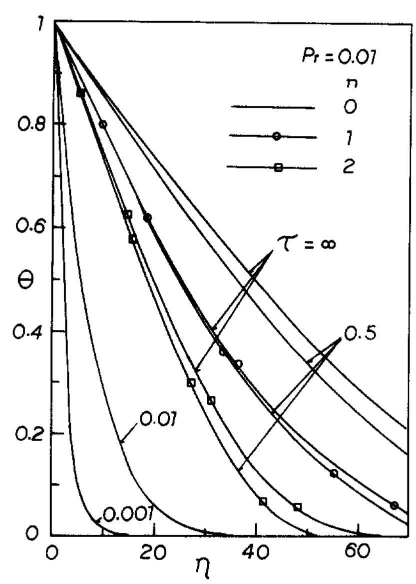

Figure 1 Transient temperature profiles for $Pr=0.01$

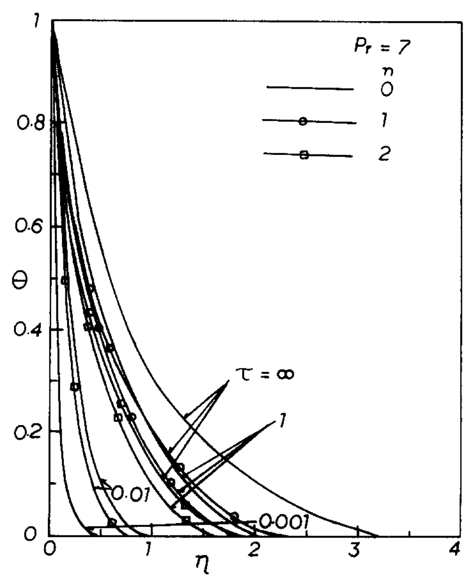

Figure 3 Transient temperature profiles for $Pr=7$

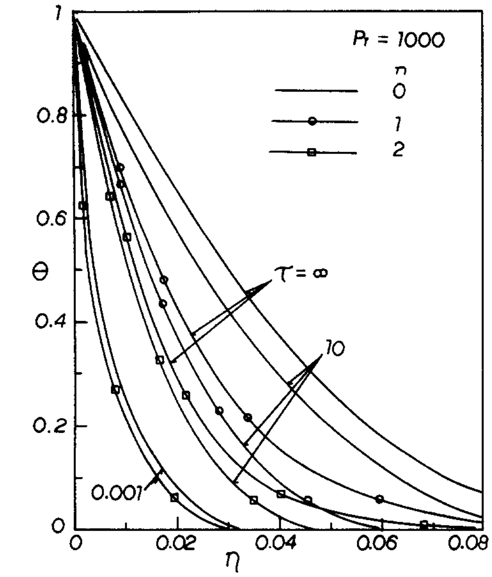

Figure 4 Transient temperature profiles for $Pr=1000$

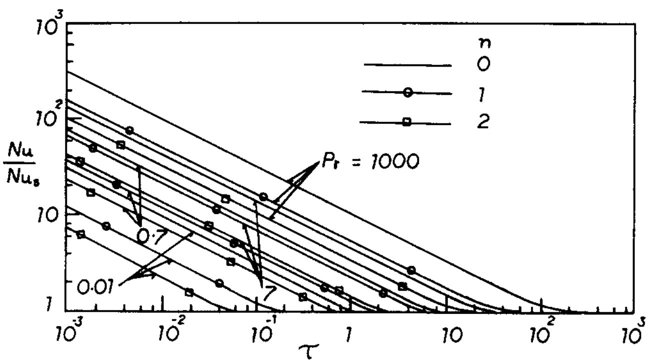

Figure 5 Ratio of instantaneous to steady state heat transfer rates

Steady-state solution at large Prandtl number

For short periods of time after the onset of process for which the thermal field has not been established, the transient solution presented above describes the thermal boundary development and rate of heat transfer. However, at longer times the process reaches steady-state. For steady-state, the momentum equation, energy equation and the boundary conditions are essentially the same as in the transient case except the left hand side of Equation 2 is 0. For large Prandtl number fluids, the heat transfer takes place within a very thin boundary layer that lies well within the hydrodynamic boundary layer and the thickness of the thermal boundary layer is proportional to $Pr^{-1/4}$.[23] Here we express the θ by an asymptotic series in descending powers of Pr

$$\theta=\theta_0+\theta_1 Pr^{-1/2}+\theta_2 Pr^{-1}+\theta_3 Pr^{-3/2}+\cdots \quad (3)$$

In Equation 3 θ_0, θ_1, and θ_2 are called zero, first, and second order solutions, respectively. Using an expression for stream function[17] which best fits the solution of the momentum equation

$$f=1.61605[1-e^{-\eta}(1+0.381207\eta+0.0185019\eta^2+0.0054350\eta^3)] \quad (4)$$

and the expression for θ given by Equation 3, the following set of equations is obtained from steady-state energy equation for large Prandtl number for solution up to second order in θ

$$\frac{d^2\theta_0}{d\zeta^2}+0.5\zeta\frac{d\theta_0}{d\zeta}-n\theta_0=0 \quad (5)$$

$$\frac{d^2\theta_1}{d\zeta^2}-0.110937\zeta^2\frac{d\theta_0}{d\zeta}+0.5\zeta\frac{d\theta_1}{d\zeta}+0.443748n\zeta\theta_0-n\theta_1=0 \quad (6)$$

$$\frac{d^2\theta_2}{d\zeta^2}-0.110937\zeta^2\frac{d\theta_1}{d\zeta}+0.5\zeta\frac{d\theta_2}{d\zeta}+0.443748n\zeta\theta_1-n\theta_2=0 \quad (7)$$

where stretched coordinate $\zeta=Pr^{1/2}\eta$. Equations 5–7 are solved together with boundary conditions: at $\zeta=0$, $\theta_0=1$, $\theta_1=\theta_2=0$, and as $\zeta\rightarrow\infty$, $\theta_0=\theta_1=\theta_2=0$, using Runge–Kutta numerical integration method. Solution of the temperature profiles was in total agreement when compared with solution obtained from transient case at $\tau=\infty$ for $Pr=1000$ (Figure 3). The heat transfer rate in terms of Nusselt number can be calculated from equation

$$Nu=-(\mathrm{Re}\,Pr)^{1/2}\left[\frac{d\theta_0(0)}{d\zeta}+\frac{d\theta_1(0)}{d\zeta}Pr^{-1/2}+\frac{d\theta_2(0)}{d\zeta}Pr^{-1}\right] \quad (8)$$

where the values of

$$\frac{d\theta_0(0)}{d\zeta},\quad \frac{d\theta_1(0)}{d\zeta}\quad\text{and}\quad\frac{d\theta_2(0)}{d\zeta}$$

are given in Table 2. An approximate integral analysis with

Table 1 Values of $-\theta_s'(0)$

$n=Pr$	0	(c)	1	2
0.01	0.011384	0.008134	0.029770	0.048279
0.1	0.073875	0.073003	0.199627	0.306778
0.7	0.35107	0.35015	0.80244	1.12149
(d)	0.3508		0.8028	1.1211
1	0.44357	0.44474	0.99250	1.37223
(a)	0.44467		0.98829	1.44262
(b)	0.53033		1.06064	1.41418
7	1.38879		2.86395	4.00511
(a)	1.39000		2.86301	4.06852
(b)	1.40312		2.80624	3.74072
10	1.68092	1.68063	3.44852	4.63916
(d)	1.68080		3.46150	4.64310
(a)	1.68308		3.44719	4.88843
(b)	1.67702		3.35038	4.47205
100	5.54563	5.54471	11.17599	15.51914
(a)	5.54671		11.16613	15.72236
(b)	5.30319		10.60638	14.14186
1000	17.79246	17.74612	35.53483	49.77494
(a)	17.77482		35.53596	49.96251
(b)	16.77019		33.54038	44.72050

(a) asymptotic steady-state solution
(b) approximate integral solution
(c) Ref. 18
(d) Ref. 15

Table 2

$n=$	0	1	2
$\frac{d\theta_0(0)}{d\zeta}$	−0.564192	−1.127165	−1.583412
$\frac{d\theta_1(0)}{d\zeta}$	0.092507	0.107222	0.108635
$\frac{d\theta_2(0)}{d\zeta}$	0.027003	0.031648	0.032161

superposition technique using Duhamel's theorem (for method see e.g., Ref. 24) was carried out, assuming third degree polynomials in velocity and temperature profiles. At large Prandtl number for which the assumption that the thermal boundary layer thickness is very small compared with momentum boundary layer thickness is valid, an approximate solution for Nusselt number was obtained as

$$Nu = 0.53033(\mathrm{Re}\,Pr)^{1/2} \quad \text{for } n=0 \tag{9a}$$

$$Nu = 0.53033(\mathrm{Re}\,Pr)^{1/2} n\beta(n, 1/2) \quad \text{for } n>0 \tag{9b}$$

where β is a beta function and the values of which are available from mathematical tables.[25] The Nusselt numbers calculated with Equations 9a,b and the asymptotic solution are compared with the numerical results in Table 1. Assuming the numerical solution as exact solution, the error in prediction of Nusselt number using approximate integral solution for $Pr=1000$ at $n=2$ is 10.15%, and at $Pr=1$ and $n=0$ the error is 19.5%. The Nusselt number calculated with asymptotic solution shows an error of 5.1% at $Pr=1$ and $n=2$. The errors shown here identify maximum errors for the range of parameters studied. Hence, the prediction of heat transfer rates can be said to be valid for the range of Prandtl number from 1 to 1000, with errors less than 20% and 6% respectively from approximate integral solution and asymptotic solution. The prediction of Nusselt number from the asymptotic steady-state solution is valid for Prandtl number 7 to ∞, with error less than 2%. Though the prediction of heat/mass transfer rates from approximate integral solution has large error (20%), it is still useful analytical expression in estimation of heat/mass transfer to a continuously moving plate whose surface temperature/concentration can be represented by power law variation with distance.

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